Science Olympiad
BirdSO In-Person Invitational

January 13, 2024

Astronomy C

Directions:

- Each team will be given **50 minutes** to complete the test.
- There are three sections: **§A** (Qualitative), **§B** (Deep-Sky Objects), and **§C** (Quantitative).
- For calculation questions, round your answers to **2 decimal places** unless otherwise specified.
- The use of AI tools (i.e. ChatGPT) are expressly forbidden.
- Tiebreakers, in order: §C, §C1, §B, §A-I, §A1, . . . , §A18.
- Best of luck! And may the odds be ever in your favor.

Written by:
Jimmy Ragan, *jragan@caltech.edu*
Phoenix He, *phx@ucla.edu*
Robert Lee, *robertyl@ucla.edu*

Feedback? **Test Code**: *2024BirdSOIP-AstronomyC-Eccentric*

Section A: Qualitative

Use the image in Image Set A to answer the following questions. This section contains a total of 50 points.

A-I: General Knowledge

There are 18 questions in this subsection. Unless otherwise specified, each question is worth two points, for a total of 33 points.

1. [1 pt] What element are main sequence stars primarily composed of?

   A. Hydrogen
   B. Helium
   C. Carbon
   D. Oxygen

2. [1 pt] From lowest to highest temperature, what are the spectral classifications found on a Hertzsprung-Russell (H-R) diagram?

   A. OBAFGKM
   B. OBAFKGM
   C. MKGFABO
   D. MGKFABO

3. What phenomenon primarily causes reflection nebulae to appear blue?

   A. Absorption of red and NIR photons by interstellar dust
   B. Limb darkening
   C. Rayleigh scattering
   D. Thomson scattering

4. In addition to the Kelvin-Helmholtz mechanism, pre-main sequence (PMS) stars can also produce energy from what element through the proton-proton III chain?

   A. Helium
   B. Lithium
   C. Beryllium
   D. Barium

5. [3 pts] In the classical proton-proton chain shown below, which of the following quantities are conserved? (Select all that apply)

$$^1_1\text{H} + ^1_1\text{H} \rightarrow ^2_1\text{H} + e^+ + \nu_e$$
$$^2_1\text{H} + ^1_1\text{H} \rightarrow ^3_2\text{He} + \gamma$$
$$^3_2\text{He} + ^3_2\text{He} \rightarrow ^4_2\text{He} + 2\,^1_1\text{H}$$

   A. Electric charge
   B. Lepton number
   C. Isospin
   D. Baryon number

6. Single-progenitor white dwarfs may collapse into neutron stars if it exceeds the Chandrasekhar mass. During this process, protons are crushed under extreme pressure and absorb free electrons in a white dwarf's atmosphere. What is this process called?

   A. Beta minus decay
   B. Inverse beta decay
   C. Beta plus decay
   D. Proton decay

7. Which of the following statements are true about Herbig-Haro objects?

   A. The ejected gas can only flow in a turbulent manner.
   B. Terminal working surfaces arise when shockwaves from radially-expanding gas collide with the interstellar medium.
   C. Internal working surfaces arise when fast jets collide with the interstellar medium, creating shockwaves.
   D. The direction of gas outflow from Herbig-Haro objects is perpendicular to the star's rotational axis.

8. [1 pt] Stars on the Hayashi track have opacities mediated by H^+ ions. What mode of heat transport is employed by such stars?

   A. Advection
   B. Conduction
   C. Convection
   D. Radiation

9. The pulsations in T Tauri stars are driven by opacity in the stellar envelope due to what ion?

   A. H^+
   B. He^+
   C. He^{2+}
   D. Li^+

Please refer to Image A1 for the following six (6) questions that follow the evolutionary stages of a $1.1\,M_\odot$ star. Answer using letters A-G, each letter can be used multiple times.

10. On which evolutionary branch(es) would the CNO cycle occur?

11. Where would the star be in hydrostatic equilibrium?

12. Where would the star leave the main sequence?

13. Which stage(s) have stellar interiors composed partly or entirely of degenerate matter?

14. In which stage(s) would the star undergo mass loss as a result of surface gravity being overcome by radiation pressure?

15. Which stage of stellar evolution, characterized by the onset of helium fusion in the stellar core, is preceded by the helium flash?

16. Which of the following effects can dim a star's intrinsic brightness in the visible spectrum? (Select all that apply)

   A. Presence of hundreds of dim, overlapping absorption lines.
   B. Interstellar extinction between the object and the observer.
   C. Increase in radius of the stellar envelope.
   D. Gravitational lensing by a high-mass object between the object and the observer.

17. Collision broadening is apparent in spectral lines as a result of collisions between particles that follow the Maxwell-Boltzmann distribution. Collision broadening is more apparent in red dwarfs compared to red giants, what can you conclude about red giants compared to red dwarfs?

   A. Red dwarfs have a higher B-V index than red giants.
   B. Red dwarfs have lower angular velocities than red giants.
   C. The stellar atmospheres of red dwarfs have higher pressures than red giants.
   D. Red dwarfs form mainly in quiescent galaxies while red giants form in galaxies with supermassive black holes at its center.

18. [1 pt] C-N and C-O make up the majority of white dwarfs, which are remnants of AGB stars that have lost their stellar envelope. The carbon was created through what process?

   A. Proton-proton chain
   B. Nuclear statistical burning
   C. Slow neutron capture
   D. Triple-alpha process

A-II: JS9 Investigation

There are 9 questions in this subsection. Points are shown for each question or sub-question, for a total of 17 points.

Setup Instructions

- Navigate to birdsoip-24-js9.tiiny.site
- Select `[File > Open Remote]` and open the image with the CORS server using the Dropbox link.
- Finally, you may change the color map if you'd like.

19. [2 pts] Qualitatively describe the object on the screen.

20. Getting to know our mystery object... *(Hint: The FITS header contains valuable information about the observation.)*
    (a) [1 pt] What is the name of this object?
    (b) [2 pts] What survey took this image?
    (c) [2 pts] In what wavelength regime was this image taken?

21. [2 pts] What is the angular diameter of the bright central object? Please give your answer in arcseconds. *(Hint: The FITS observation uses 1 arcsecond = 1.5 pixels.)*

22. [2 pts] Select a possible set of coordinates for the largest cloud mass on the right of the central object.
    A. 17:58:14.429, +66:38:08.80
    B. 17:59:07.449, +66:41:01.64
    C. 17:59:05.671, +66:34:47.67
    D. 17:58:04.912, +66:34:45.46

The following three (3) questions can be answered without completing the questions above.

23. [2 pts] Would you expect the clouds adjacent to the central object to consist of H I or H II? Explain your answer.

24. [2 pts] Consider Images A2 and A3, which are taken in different wavelengths compared to the FITS file you investigated. Which area has the highest bolometric luminosity?
    A. Region A B. Region B C. Region C

25. [2 pts] What could the object in the center be, and what is this object overall?
    A. Protostar; debris disk
    B. Stellar-mass black hole; starburst galaxy
    C. T Tauri star; Herbig-Haro object
    D. White dwarf; planetary nebula

Section B: Deep-Sky Objects

Use the images in Image Set B to answer the following questions. Unless otherwise specified, each question is worth two points, for a total of 60 points.

1. [1 pt] Identify the image that depicts HR 8799.
   A. Image B6
   B. Image B9
   C. Image B10
   D. Image B11

2. In which constellation is HR 8799 located?
   A. Bootes
   B. Pegasus
   C. Dorado
   D. Lyra

3. If the planets e, d, and c of HR 8799 are in a 4:2:1 resonance, how does that affect the structure of the system?
   A. Speeds up the rate of planetary migration
   B. Decreases the orbital period over time
   C. Increases the stability of the orbits
   D. Forms stable belts of asteroids and planetesimals between them

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4. [1 pt] Which image depicts NGC 1333?
   A. Image B5
   B. Image B7
   C. Image B8
   D. Image B9

5. What mass does the nebula approximately contain?
   A. $50\,M_\odot$
   B. $500\,M_\odot$
   C. $5000\,M_\odot$
   D. $50\,000\,M_\odot$

6. Many of the young stars born in NGC 1333 are not visible in the optical wavelength. Why?
   A. New stars are often obscured within the dusty cloud they formed from.
   B. Dense gas strongly polarizes light from 300–800 nm making them hard to detect.
   C. Protostars do not emit a lot of visible light, but primarily in infrared.
   D. The high speed jets from the stars heavily redshift the visible light.

7. The embedded stellar clusters in NGC 1333 follow what morphology?
   A. Turbulent
   B. Hierarchical
   C. Centralized
   D. Distributed

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8. How many planets of TRAPPIST-1 are located in the habitable zone?
   A. 0 B. 1 C. 3 D. 7

9. What method would be most applicable for creating a dynamical model of TRAPPIST-1 from transit-timing data?
   A. Machine learning
   B. Smoothed-particle hydrodynamics
   C. Bayesian inference
   D. N-body

10. Planet formation follows three stages of growth: (1) dust grains condensed from cooling gas stick together by ______, (2) planetesimals form, and (3) planetesimals 'coagulate' through processes dominated by ______.
    A. hydrogen bonding; accretion
    B. electrostatic forces; accretion
    C. van der Waals forces; gravity
    D. hydrogen bonding; gravity

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11. [1 pt] Which image depicts HD 169142?
    A. Image B6
    B. Image B9
    C. Image B10
    D. Image B11

12. Which of the following is **not** a piece of evidence not used to support the discovery of an exoplanet around HD 169142?
    A. A wake consistent with hydrodynamical simulations is found trailing the protoplanet.
    B. The exoplanet candidate is located in an annular gap of the protoplanetary disk.
    C. The motion of the exoplanet candidate closely fits Keplerian predictions.
    D. A radial velocity curve of the host star matches perturbations by a Jovian-mass protoplanet.

13. What chemical signature was detected—for the first time in association with a protoplanet—at HD 169142 b?
    A. Silicon monosulfide (SiS)
    B. Carbon monoxide (CO)
    C. Cyanoacetylene (HC_3N)
    D. Sulfur monoxide (SO)

14. The molecule in the previous question is formed through what physical process?
    A. Released in shocks
    B. MHD-driven disk winds
    C. Sublimation of volatile-rich ice
    D. Outflow from jets on the planet surface

15. The light curve of V1298 Tauri is shown in Image B1. What is this variability attributed to?
    A. Radial pulsation
    B. Stellar rotation
    C. Transits and occultations
    D. Axisymmetric distribution of star spots

16. The orbit of what planet can encircle the region in which the four exoplanets of V1298 Tauri inhabit?
    A. Mercury
    B. Venus
    C. Earth
    D. Saturn

17. [3 pts] You are an astronomer validating the existence of V1298 Tauri b. You have the exact mass of V1298 Tauri, orbit period of V1298 Tauri b, and a perfect (i.e. no noise or error) light curve of a transit. However, you overestimate the distance to the system and assume (incorrectly) the impact parameter of the exoplanet is 0. How would your estimate of the host star's temperature $\widetilde{T}_\star$ and exoplanet radius $\widetilde{R}_\text{p}$ compare to the real values $T_\star$ and R_p? *(Hint: You don't have the stellar spectrum to use Wien's law! What other formula can you use?)*
    A. $\widetilde{T}_\star > T_\star$, $\widetilde{R}_\text{p} > R_\text{p}$
    B. $\widetilde{T}_\star > T_\star$, $\widetilde{R}_\text{p} < R_\text{p}$
    C. $\widetilde{T}_\star < T_\star$, $\widetilde{R}_\text{p} > R_\text{p}$
    D. $\widetilde{T}_\star < T_\star$, $\widetilde{R}_\text{p} < R_\text{p}$

18. [1 pt] Which image taken by the Chandra X-ray Observatory depicts WASP-18, the host star to WASP-18b?
    A. Image B6
    B. Image B7
    C. Image B10
    D. Image B11

19. [3 pts] Chandra detected no X-ray emission from WASP-18 even though it is a relatively young star. How is this reconciled?
    A. Large number of sunspots decrease the rate of radiative transfer out from the stellar core.
    B. An excess of synchrotron radiation at the stellar poles redirects the X-ray emission away.
    C. Tidal forces from its massive exoplanet attenuate the convection within the star.
    D. The high abundance of lithium dampens the star's magnetic fields.

20. WASP-18b was discovered in 2009 using which two detection methods in tandem? (Select two)
    A. Transit
    B. Microlensing
    C. Direct imaging
    D. Radial velocity

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21. What type of object is TW Hya?
    A. Main sequence star
    B. T Tauri star
    C. Herbig Ae/Be star
    D. Brown dwarf

22. Identify the function, $v(r)$, the rotation curve of TW Hya's disk would match closest to if it were Keplerian, where r is radius and a, b, and c are arbitrary positive constants. *(Hint: Set gravitational and centripetal force equal and solve for velocity as a function of radius.)*
    A. ar
    B. $ar/(b + cr^2)$
    C. $a/\sqrt{r}$
    D. $a(1 - e^{-br})$

23. Image B2 (Teague et al. 2019) depicts the map of the ^{12}CO brightness temperature, a proxy of the local gas temperature. This is calculated from the emission line intensity under the assumption it is optically thick. What does it mean for it to be optically thick?
    A. The density of the medium is extremely high which increases the collision frequency.
    B. The local gas surface has high albedo for infrared and longer wavelengths.
    C. The transmissivity of the disk is high in the EHF band ^{12}CO emits at (0.87 mm).
    D. The gas has high absorptivity at the emission line wavelength.

24. [1 pt] Teague et al. (2019) subtracts an azimuthally averaged radial profile (i.e. average over a circle with a fixed radius) to form residuals. They are plotted in Image B3 and reveal spiral substructures which have been indicated by linear spirals. If the disk is a trailing spiral, what direction does it rotate?
    A. Clockwise
    B. Counterclockwise

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25. [1 pt] Which object in the Luhman 16 system is shown in Image B4?

A. Luhman 16A B. Luhman 16B

26. What is the rotational period of the object?

A. ∼3.1 hours B. ∼4.5 hours C. ∼5.1 hours D. ∼5.5 hours

27. What instrument of the Very Large Telescope(VLT) was used to make this surface map?

A. MUSE (Multi-Unit Spectroscopic Explorer)

B. CRIRES (CRyogenic InfraRed Echelle Spectrograph)

C. ISAAC (Infrared Spectrometer And Array Camera)

D. FORS2 (FOcal Reducer and low dispersion Spectrograph)

28. [3 pts] Luhman 16 is the third-closest star system after Proxima Centauri and Barnard's star. Despite its proximity, this system was discovered only in 2011. Why is this so?

A. Luhman 16 is the first extragalactic rogue brown dwarf to be discovered, and its radial velocity is too fast to be detected previously.

B. Luhman 16 was previously thought to be a population II star because it resides in the galactic halo.

C. Luhman 16 was found within a starburst galaxy, and the infrared excess from young stars obscures the radiation emitted by the star system.

D. Luhman 16 resides in the galactic thin disk, which is populated with luminous objects, which makes the photometric discovery of the optically-dim system difficult.

29. [3 pts] Why are some areas on the map darker, and other areas brighter?

A. The brighter areas have thicker clouds with higher albedo, reflecting more light to be detected by the observer.

B. The darker areas indicate sooty carbon monoxide clouds while the brighter areas indicate diatomic nitrogen clouds.

C. The darker and brighter areas are contracting and expanding, respectively, due to positive and negative gravitational anomalies because brown dwarfs are not yet in hydrostatic equilibrium.

D. The darker areas have thicker cloud cover that occult the hotter interior of the brown dwarf.

30. How does Doppler spectroscopy help make this map?

A. Spectral data show blue-shifted and red-shifted spectral lines, which indicate the dwarf's atmosphere is expanding and contracting, respectively.

B. Thicker emission lines indicate areas of high, which corresponds to the dark spots on the map, and vice versa.

C. Doppler shift can tell whether the dark spots are moving towards or away from the observer, from which the observer can acquire the direction of rotation.

D. Infrared spectroscopy, rather than doppler spectroscopy, was used to make this map.

31. (Short Answer) How is understanding the stellar atmospheres of brown dwarfs, such as Luhman 16AB, significant to exoplanet research?

Section C: Quantitative

In this section, you will be asked to perform calculations and provide numerical answers, as well as answer qualitative follow up questions. Please round your answers to 2 decimal places and provide your answers in the specified units. Answers in the wrong units, or with no units provided, will not receive any points. Double check and box your final answer! Work will not be graded (and you are not required to show it). More scratch paper is available if needed. This section contains a total of 40 points.

Conversions you may find helpful:
$1\,\text{au} = 1.496 \times 10^{11}\,\text{m}^*$
$1\,\text{ly} = 9.461 \times 10^{15}\,\text{m}$
$1\,\text{pc} = 3.086 \times 10^{16}\,\text{m}$
$1\,\text{M}_\odot = 1.989 \times 10^{30}\,\text{kg}$ (Solar Mass)
$1\,\text{M}_{♃} = 1.898 \times 10^{27}\,\text{kg}$ (Jupiter Mass)
$1\,\text{R}_\odot = 6.957 \times 10^{8}\,\text{m}$ (Solar Radius)
$1\,\text{R}_\oplus = 6.371 \times 10^{6}\,\text{m}$ (Earth Radius)
$G = 6.6743 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$

*Note: There was a typo on the test where the erroneous values of 1.469×10^{11} m was indicated. The answer key was updated to carry through this error on affected questions, so using either value to solve the problem was graded as correct.

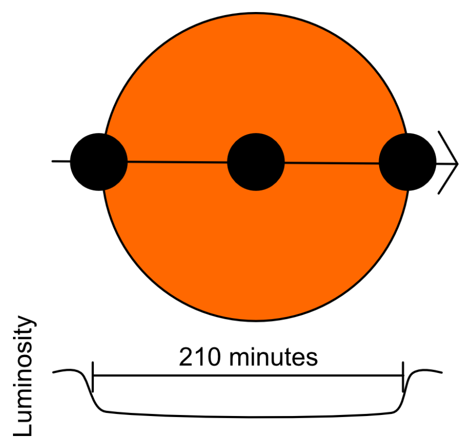
1. You are working on a catalog of nearby stars and exoplanets and are trying to determine various properties, using observational measurements. Each subpart of this question can be answered independently from each other.

   (a) [2 pts] You're studying a K5 dwarf star, which you know typically has an absolute magnitude of 6.6. From your observations, it has an apparent magnitude of about 10.3. How far away (in parsecs) is the star?

   (b) [2 pts] For another star, you use a spectrograph to measure the intensity of the star at different wavelengths. You note a peak in intensity at a wavelength of 450 nm. Assuming the star is a black body, what temperature is it (in Kelvin)?

   (c) [2 pts] A candidate habitable exoplanet is discovered to have a radius 1.5 times that of Earth. Assuming the planet has the same density as Earth, what would be the surface acceleration due to gravity, in terms of Earth's gravity, g? (For example, the acceleration due to gravity on the surface of Earth is 1 g. On the moon, it is weaker at 0.166 g)

   (d) [3 pts] Direct imaging is becoming more popular for discovering exoplanets. However, these planets are hard to resolve, because of their miniscule angular size. At a relatively close distance of 15 pc (relatively close in an astronomical sense that is!), how big would a planet the same size as Earth appear, in microarcseconds (10^{-6} arcseconds)?



   (e) [3 pts] An exoplanet is discovered that transits its host star every 18.5 days. The host star is known to have a size of $0.6\,R_\odot$. The transit takes 210 minutes to complete, measuring the full-width half maximum of the light curve, which we can take as a good approximation of the time for the center of the planet to transit the star (see the diagram above). Assuming this is an equatorial transit and a circular orbit, what is the radius of this exoplanet's orbit, in au? **Please give 3 decimal places for this question.**

2. The following subparts all refer to the same system where an exoplanet has been detected using the radial velocity method. From other observations, you can determine the star has a mass of $0.7\,M_\odot$. Assume circular orbits and no other objects in this system for all subparts of this question.

   (a) [4 pts] The star's radial velocity is observed to vary with an amplitude of $262\,\mathrm{m\,s^{-1}}$ ($514\,\mathrm{m\,s^{-1}}$ peak-to-peak) on a period of 9 days. What is the minimum mass of the planet, in Jupiter Masses ($M_♃$)? *(Note: Some of the conversions above may be helpful.)*

   (b) [2 pts] Why is your previous answer a minimum mass?

   (c) [2 pts] What type of exoplanet is this most likely to be?

   (d) [3 pts] Shortly after your discovery, a team performing direct imaging of exoplanets is able to capture images of your exoplanet, and determines the orbit has an angular radius of 15 microarcseconds (15×10^{-6} arcseconds). How far away is this system (in parsecs)?

3. The following subparts all consider another exoplanet, discovered by the transit method. The host star of this system is Sun-like, with a size of $0.85\,R_\odot$. We will determine in this question if this planet might be habitable. Assume circular orbits and no other objects in this system for all subparts of this question.

   (a) [3 pts] This exoplanet is determined to completely transit its host star, reducing the luminosity by a maximum of 2024 parts per million (ppm). What is the size of this planet in Earth Radii?

   (b) [3 pts] From your previous calculation, what type of exoplanet is this most likely to be? Would this type of exoplanet be conducive for Earth-like life? Identify one reason why or why not.

   (c) [4 pts] From the period of the transit and estimates of the system mass, you are able to calculate that the planet orbits the host star at a radius of 0.63 au. Assume the host star is a black body with a temperature of 5300 K. What is the solar flux at the radius of this planet's orbit (in $\mathrm{W\,m^{-2}}$)?

   (d) [5 pts] Without a direct image of the planet, we are unsure of the composition or its appearance, so we will assume an albedo of 0.29 (matching Earth's) for our analysis. With this and the radius you computed earlier, what would be the radiative equilibrium temperature of this planet, in Kelvin?

   (e) [2 pts] From the result of your previous, is this planet habitable? Why or why not?

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BirdSO In-Person Invitational
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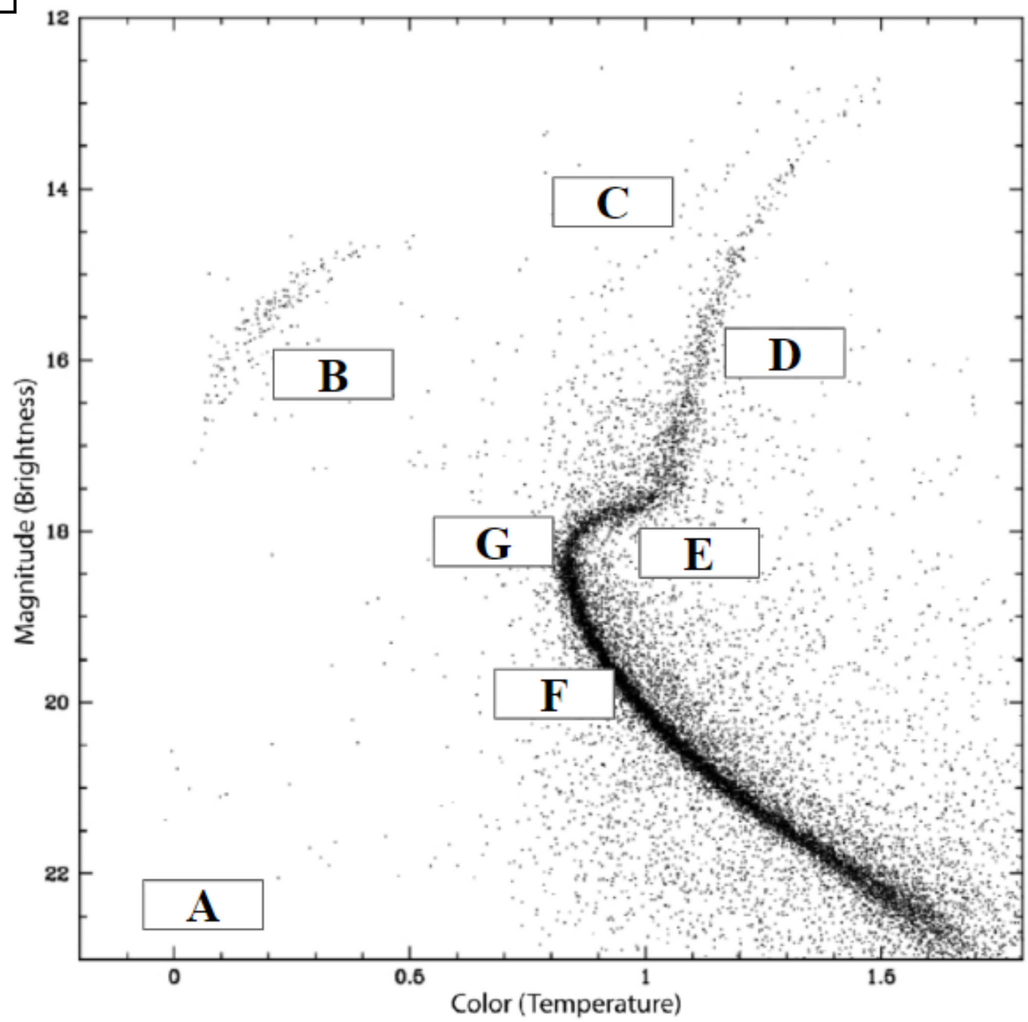
Astronomy C Image Sheet

Directions:

- Do not open until the test begins.

Image Set A

1



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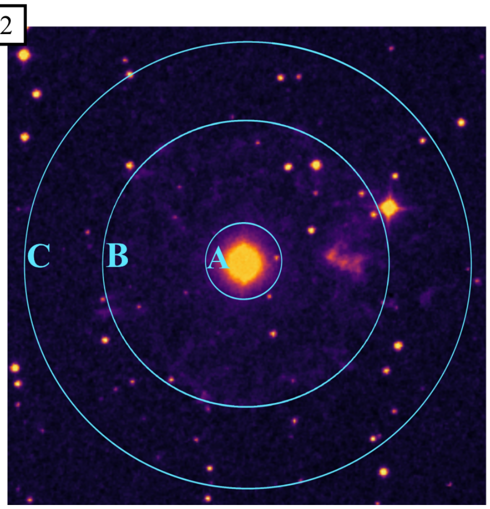

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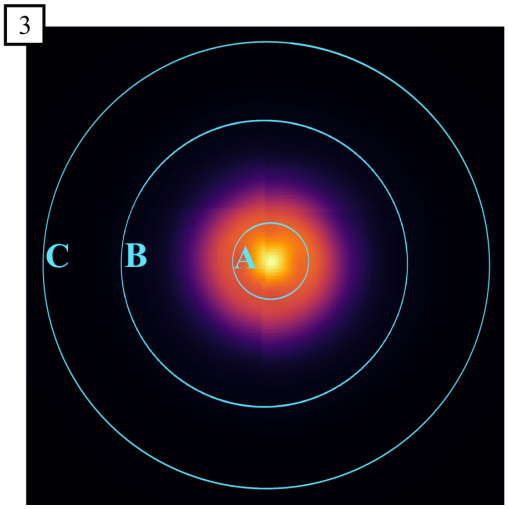
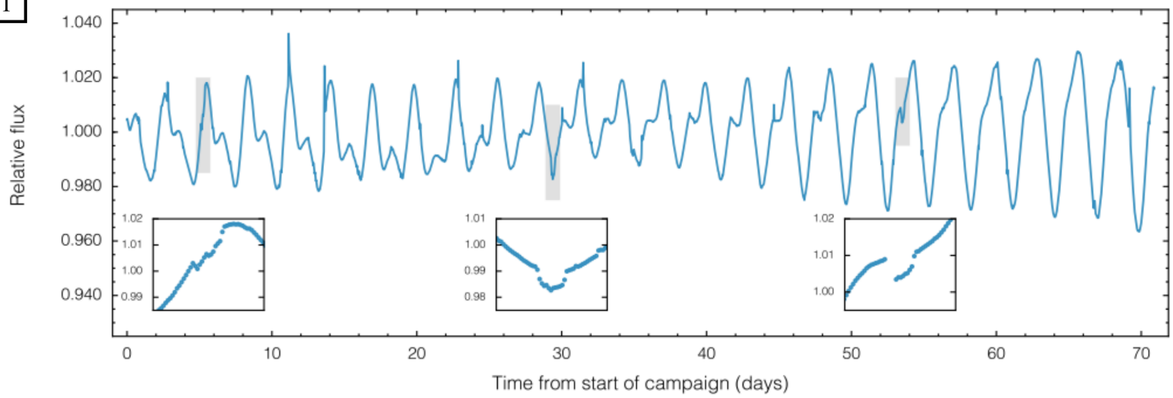


Image Set B

1



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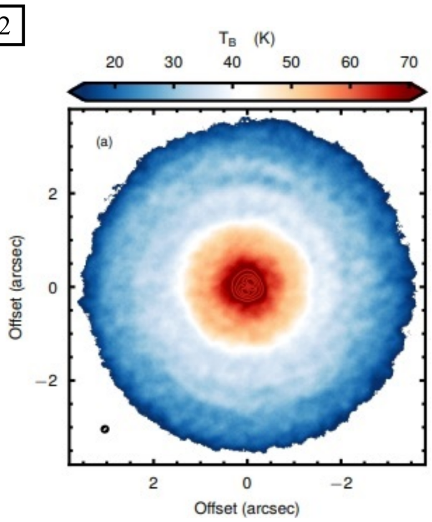

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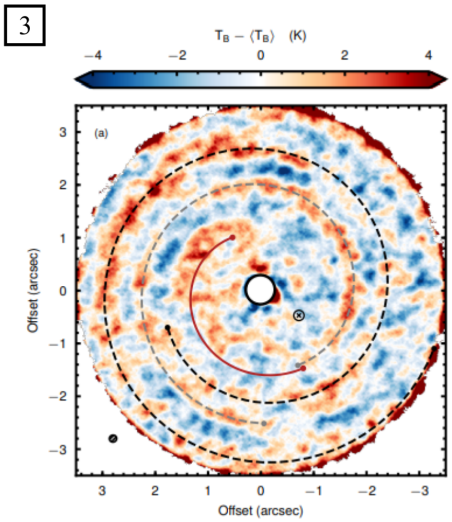

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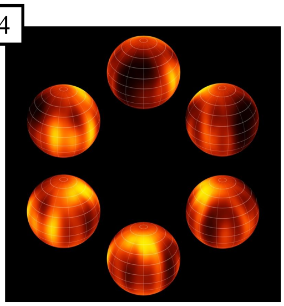

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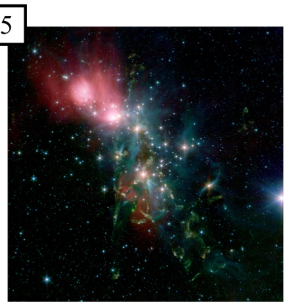

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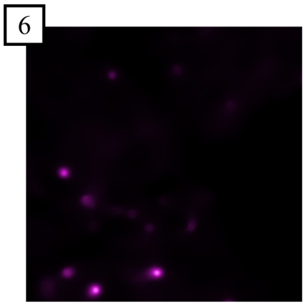

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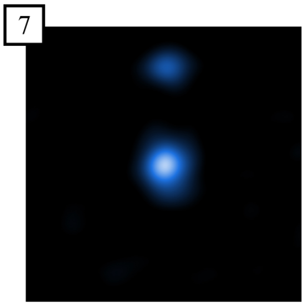

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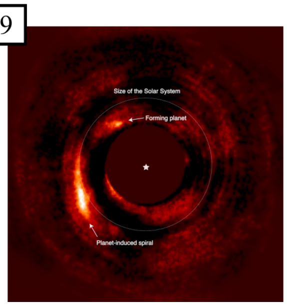

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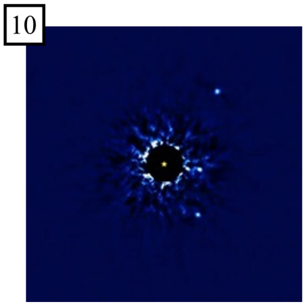

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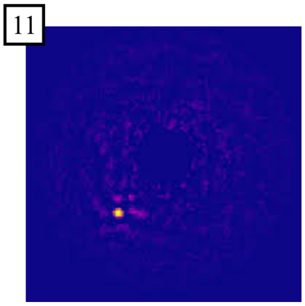

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Astronomy C Answer Sheet

Team Name and Number: ______________________________

Participant Name(s): ______________________________

Total Score: ____ / 150 **Rank:** ____

Directions:

- Read the directions on the test cover.

Section A (50 points)

1. ______ 2. ______ 3. ______ 4. ______ 5. ______

6. ______ 7. ______ 8. ______ 9. ______ 10. ______

11. ______ 12. ______ 13. ______ 14. ______ 15. ______

16. ______ 17. ______ 18. ______

19. ______

20. (a) ______
(b) ______
(c) ______

21. ______

22. ______

23. ______

24. ______

25. ______

Section B (60 points)

1. ______ 2. ______ 3. ______ 4. ______ 5. ______

6. ______ 7. ______ 8. ______ 9. ______ 10. ______

11. ______ 12. ______ 13. ______ 14. ______ 15. ______

16. ______ 17. ______ 18. ______ 19. ______ 20. ______

21. ______ 22. ______ 23. ______ 24. ______ 25. ______

26. ______ 27. ______ 28. ______ 29. ______ 30. ______

31. ______

Section C (40 points)

1. (a)

(b)

(c)

(d)

(e)

2. (a)

(b)

(c)

(d)

3. (a)

(b)

(c)

(d)

(e)

Science Olympiad
BirdSO In-Person Invitational
January 13, 2024

Astronomy C Solutions

Section C Solutions

Section C: Quantitative

In this section, you will be asked to perform calculations and provide numerical answers, as well as answer qualitative follow up questions. Please round your answers to 2 decimal places and provide your answers in the specified units. Answers in the wrong units, or with no units provided, will not receive any points. Double check and box your final answer! Work will not be graded (and you are not required to show it). More scratch paper is available if needed. This section contains a total of 40 points.

Conversions you may find helpful:
$1\,\mathrm{au} = 1.496 \times 10^{11}\,\mathrm{m}^*$
$1\,\mathrm{ly} = 9.461 \times 10^{15}\,\mathrm{m}$
$1\,\mathrm{pc} = 3.086 \times 10^{16}\,\mathrm{m}$
$1\,\mathrm{M}_\odot = 1.989 \times 10^{30}\,\mathrm{kg}$ (Solar Mass)
$1\,\mathrm{M}_{♃} = 1.898 \times 10^{27}\,\mathrm{kg}$ (Jupiter Mass)
$1\,\mathrm{R}_\odot = 6.957 \times 10^{8}\,\mathrm{m}$ (Solar Radius)
$1\,\mathrm{R}_\oplus = 6.371 \times 10^{6}\,\mathrm{m}$ (Earth Radius)
$G = 6.6743 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$

*Note: There was a typo on the test where the erroneous values of 1.469×10^{11} m was indicated. The answer key was updated to carry through this error on affected questions, so using either value to solve the problem was graded as correct.

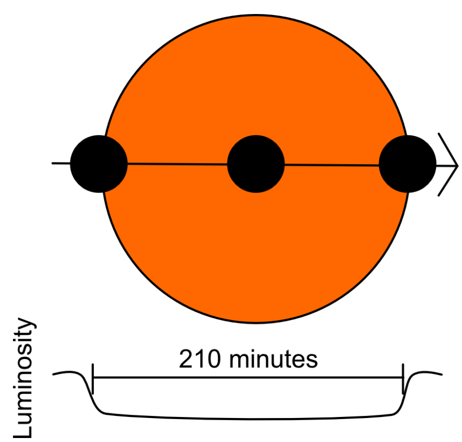
1. You are working on a catalog of nearby stars and exoplanets and are trying to determine various properties, using observational measurements. Each subpart of this question can be answered independently from each other.

   (a) [2 pts] You're studying a K5 dwarf star, which you know typically has an absolute magnitude of 6.6. From your observations, it has an apparent magnitude of about 10.3. How far away (in parsecs) is the star?

   (b) [2 pts] For another star, you use a spectrograph to measure the intensity of the star at different wavelengths. You note a peak in intensity at a wavelength of 450 nm. Assuming the star is a black body, what temperature is it (in Kelvin)?

   (c) [2 pts] A candidate habitable exoplanet is discovered to have a radius 1.5 times that of Earth. Assuming the planet has the same density as Earth, what would be the surface acceleration due to gravity, in terms of Earth's gravity, g? (For example, the acceleration due to gravity on the surface of Earth is 1 g. On the moon, it is weaker at 0.166 g)

   (d) [3 pts] Direct imaging is becoming more popular for discovering exoplanets. However, these planets are hard to resolve, because of their miniscule angular size. At a relatively close distance of 15 pc (relatively close in an astronomical sense that is!), how big would a planet the same size as Earth appear, in microarcseconds (10^{-6} arcseconds)?



   (e) [3 pts] An exoplanet is discovered that transits its host star every 18.5 days. The host star is known to have a size of $0.6\,R_\odot$. The transit takes 210 minutes to complete, measuring the full-width half maximum of the light curve, which we can take as a good approximation of the time for the center of the planet to transit the star (see the diagram above). Assuming this is an equatorial transit and a circular orbit, what is the radius of this exoplanet's orbit, in au? **Please give 3 decimal places for this question.**

Solution:

(a) The distance modulus formula is

$$m - M = 5\log_{10}(d) - 5,$$

where $m = 10.3$ is the apparent magnitude, $M = 6.6$ is the absolute magnitude, and d is the distance in parsecs. We use it to solve for $\boxed{d = 54.95\,\text{pc.}}$

(b) Wien's law is

$$T_{\text{eff}} = \frac{b}{\lambda_{\text{peak}}},$$

where T_{eff} is the effective temperature on the surface of the black body, $b = 2.898 \times 10^{-3}\,\text{m K}$ is the Wien's displacement constant, and λ_{peak} is the wavelength at which the spectra peaks. We use it to solve for $\boxed{T_{\text{eff}} = 6440\,\text{K.}}$

(c) Since the exoplanet has the same density of the Earth, the mass of the exoplanet scales as r^3. On the other hand, gravity scales as M/r^2. Combining we find that gravity scales as r, so the surface acceleration due to gravity is $\boxed{1.5\ g.}$

(d) We will use the angular diameter formula

$$\delta = \frac{D}{d},$$

where δ is angular diameter is in arcseconds, D is physical diameter in au, and d is distance in parsecs. Converting, we get $D = 2\,\text{R}_\oplus = 8.681 \times 10^{-5}$ au and substituting into the formula, we get $\delta = 5.787 \times 10^{-6}$ arcseconds = $\boxed{\text{5.787 microarcseconds.}}$

(e) Using the fact that this is an edge on transit to realize it takes the planet 120 minutes to go $1.2\,\text{R}_\odot = 8.348 \times 10^8$ m, so we have a velocity of $6.626 \times 10^4\,\text{m s}^{-1}$. So the orbit radius is 1.686×10^{10} m = $\boxed{0.114\,\text{au.}}$

2. The following subparts all refer to the same system where an exoplanet has been detected using the radial velocity method. From other observations, you can determine the star has a mass of $0.7\,M_\odot$. Assume circular orbits and no other objects in this system for all subparts of this question.

(a) [4 pts] The star's radial velocity is observed to vary with an amplitude of $262\,\mathrm{m\,s^{-1}}$ ($514\,\mathrm{m\,s^{-1}}$ peak-to-peak) on a period of 9 days. What is the minimum mass of the planet, in Jupiter Masses ($M_♃$)? *(Note: Some of the conversions above may be helpful.)*

(b) [2 pts] Why is your previous answer a minimum mass?

(c) [2 pts] What type of exoplanet is this most likely to be?

(d) [3 pts] Shortly after your discovery, a team performing direct imaging of exoplanets is able to capture images of your exoplanet, and determines the orbit has an angular radius of 15 microarcseconds (15×10^{-6} arcseconds). How far away is this system (in parsecs)?

Solution:

(a) We are given $262\,\mathrm{m\,s^{-1}}$ over 9 days, so we can figure out the radius of the stars orbit (around the system barycenter) from circular motion.

$$\frac{1}{2\pi} \times 262\,\mathrm{m\,s^{-1}} \times 9\,\mathrm{d} \times 86\,400\,\mathrm{s\,d^{-1}} = 3.242 \times 10^{7}\,\mathrm{m}$$

Now we can use Kepler's 3rd law,

$$\frac{MGT^2}{4\pi^2} = a^3,$$

where $M = 0.7\,M_\odot$ is the mass of the star (assuming the mass of the planet is much less), G is the gravitational constant, $T = 9\,\mathrm{d}$ is the period, and a is the semi-major axis. Assuming an edge on orbit, we find $a = 1.125 \times 10^{10}\,\mathrm{m}$ and then we use the ratio of star orbit radius to semi major axis to get the mass ratios:

$$\frac{3.242 \times 10^{7}\,\mathrm{m}}{1.125 \times 10^{10}\,\mathrm{m}} \times 0.7\,M_\odot \times \frac{1.989 \times 10^{30}\,\mathrm{kg}\,{M_\odot}^{-1}}{1.898 \times 10^{27}\,\mathrm{kg}\,{M_♃}^{-1}} = \boxed{2.114\,M_♃.}$$

(d) From previous work, we know that the semi major axis is $0.076\,625\,\mathrm{au}$. We can then directly use parallax to compute distance to the system as

$$d = \frac{r}{p} = \frac{0.076\,625\,\mathrm{au}}{15 \times 10^{-6\prime\prime}} = \boxed{5108\,\mathrm{pc.}}$$

3. The following subparts all consider another exoplanet, discovered by the transit method. The host star of this system is Sun-like, with a size of $0.85\,\mathrm{R}_\odot$. We will determine in this question if this planet might be habitable. Assume circular orbits and no other objects in this system for all subparts of this question.

(a) [3 pts] This exoplanet is determined to completely transit its host star, reducing the luminosity by a maximum of 2024 parts per million (ppm). What is the size of this planet in Earth Radii?

(b) [3 pts] From your previous calculation, what type of exoplanet is this most likely to be? Would this type of exoplanet be conducive for Earth-like life? Identify one reason why or why not.

(c) [4 pts] From the period of the transit and estimates of the system mass, you are able to calculate that the planet orbits the host star at a radius of $0.63\,\mathrm{au}$. Assume the host star is a black body with a temperature of $5300\,\mathrm{K}$. What is the solar flux at the radius of this planet's orbit (in $\mathrm{W\,m^{-2}}$)?

(d) [5 pts] Without a direct image of the planet, we are unsure of the composition or its appearance, so we will assume an albedo of 0.29 (matching Earth's) for our analysis. With this and the radius you computed earlier, what would be the radiative equilibrium temperature of this planet, in Kelvin?

(e) [2 pts] From the result of your previous, is this planet habitable? Why or why not?

Solution:

(a) Ratio of radii is found from realizing the decrease in luminosity is just the ratio of the cross-sectional areas:

$$\sqrt{2024\,\mathrm{ppm}} = 0.044\,99 \implies 0.044\,99 \times 0.85\,\mathrm{R}_\odot = \boxed{4.17\,\mathrm{R}_\oplus.}$$

(c) The Stefan–Boltzmann law is $P = A\epsilon\sigma T^4$, where P is total power (or luminosity), A is surface area, ϵ is emissivity (which is 1 for a black body), $\sigma = 5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$ is the Stefan–Boltzmann constant, and T is surface temperature. We can use it to find the luminosity of the host star and divide it by surface area of the sphere at the planet's radius:

$$\frac{4\pi \times (0.85\,\mathrm{R}_\odot \times 6.957 \times 10^{8}\,\mathrm{m})^2 \times \sigma \times (5300\,\mathrm{K})^4}{4\pi \times (0.63\,\mathrm{au} \times 1.496 \times 10^{11}\,\mathrm{m})^2} = \boxed{1761\,\mathrm{W}.}$$

(d) We need to balance the energy from the host star going into the planet with the energy leaving the planet due to its own thermal radiation. The energy in is $E_{\mathrm{in}} = I \times \pi R_{\mathrm{p}}^2 \times (1 - A_B)$, where I is the incident solar flux, R_{p} is the planet radius, and A_B is the albedo. The energy out is $E_{\mathrm{out}} = 4\pi R_{\mathrm{p}}^2 \sigma T_{\mathrm{p}}^4$, where σ is the Stefan–Boltzmann constant and T_{p} is the temperature of the planet. Setting them equal and solving for T_{p}, we get

$$\left(\frac{I \times (1 - A_B)}{4\sigma}\right)^{1/4} = \left(\frac{1761\,\mathrm{W} \times (1 - 0.29)}{4\sigma}\right)^{1/4} = \boxed{272.5\,\mathrm{K}.}$$